Reduction of Attributes in Ordinal Decision Systems

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Abstract. Rough set theory has proven to be a very useful tool in dealing with many decision situations where imprecise and inconsistent information are involved. Recently, there are attempts to extent the use of rough set theory to ordinal decision making in which decisions are made on ordering of objects through assigning them to ordinal categories. Attribute reduction is one of the problems that is studied under such ordinal decision situations. In this paper we examine some of the proposed approaches to find ordinal reducts and present a new perspective and approach to the problem based on ordinal consistency.

1 Introduction

We are often called upon to deal with data that are imprecise and inconsistent in decision situations. Many theories, such as fuzzy set theory and Dempster-Shafer theory of evidence, have been developed to handle these types of data in analysis and decision making. Rough set theory introduced by Pawlak in [7] is one of the more recent developments in this area. Since its introduction rough set theory has found to be useful in a broad spectrum of applications [3, 6]. Rough set theory acknowledges the fact that in real world situations, objects can only be identified by their known attributes and objects sharing the same attribute values become indiscernible. The indiscernible groups of objects resulting from the limited information we have about them are the basis upon which analysis and decisions must be made.

In the original rough set formulation, the attributes considered in an information system, including decision attributes, are assumed to take on nominal values. Objects sharing the same attribute values form equivalent groups which partition the object space. In such environment we may not be able to completely specify an arbitrary set of objects using the attributes available. We can approximate the set by a rough set consisting of a lower and an upper approximation, definable in the available attributes. One important area in data analysis is the selection of a suitable set of attributes among available attributes which may contain redundancy. Redundant attributes cause ineffectiveness in data analysis and induction of decision models. In rough set theory selection of attribute is studied under the topic of reducts which are minimal attribute sets with same descriptive power of the original attribute set.

D.S. Yeung et al. (Eds.): ICMLC 2005, LNAI 3930, pp. 578 – 587, 2006.

Subsequent to its introduction, rough set theory has been extended in various ways, for example, by combining it with fuzzy set theory, or replacing the equivalence relation defining the indiscernibility groups with other weaker forms of relations. In this paper we consider the formulation in which the decision is based on a set of ordered labels. In such decision situation, an ordering is created in a set of objects by assigning them ordinal class labels as in the case of assigning students A, B, C, … grades in a test. In the previous studies on application of rough set theory in ordinal decisions, the conditional attributes used to determine the decisions are also considered to be ordinal in nature [1, 2, 4, 8]. Greco et al studied the ordinal information system in the context of multicriteria decision making in [2]. They defined a reduct based on quality of approximation, using upper and lower set approximation similar to those in the classic rough set model. We will describe this system in greater details in Section 3 to contrast it with our approach. Bioch and Popova [1] studied reducts in monotone information systems by examining the monotone discernibility matrix, which corresponds to discernibility matrix in classic rough set, in finding monotone reducts. In [8], the authors examine the concept of reduct by considering the partitioning of binary relation space by ordering expressions in the set of attributes concerned. We have also studied this problem by focusing on the approximation of the implicit decision ordering using the available ordinal attributes in [4]. In this paper, we examine the situation where the ordinal decision is made based on condition attributes which take nominal values. Certainly nominal attributes are common, and in some situations like absence of monotonic relation, it may be more effective in decision analysis to treat ordinal attributes as nominal. We will examine reducts required for approximation of the decision ordering in this environment. We formulate a new type of discernibility matrix which we use to find ordinal reducts and illustrate this with an example.

In the following, we first present an overview of rough set in Section 2. We discuss ordinal information systems in Section 3. In Section 4 we examine the approximation of decision ordering and the associated concept of ordinal reducts. Section 5 introduces the new definition of ordinal separability and ordinal discernibility matrix and how it can be used to find ordinal reducts. The last section concludes with some of our ongoing investigation directions.

2 Rough Sets Overview

In this section we recap some of the basic formulation of rough set theory and related concepts we used in this paper. More details can be found in the tutorial paper [3]. An information system is a pair $S=(U,A)$ where U is the universe of discourse with a finite number of objects, and A is a set of attributes defined on U. Each attribute a of A is a function $a:U \to V_a$ which maps objects of U to the value set V_a. A decision system is a special information system $\Delta=(U,A\cup\{d\})$ in which there is a distinguished attribute d called the decision attribute with corresponding decision value set V_d. For any subset of attributes $B \subseteq A$, an indiscernibility relation $I_B=\{(x,y):a(x)=a(y)\forall a\in B\}$ is generated which partitions U into equivalence classes U/I_B of objects indistinguishable with respect to attributes in B. The equivalence class containing $x \in U$ can be defined by $[x]_B = \{y : a(x) = a(y) \forall a \in B\}$.

For any subset $X \subseteq U$ and $B \subseteq A$, X can be approximated by a rough set consisting of a lower and an upper approximation defined respectively as:

$$\underline{B}X := \{x : [x]_B \subseteq X\} \tag{1}$$

$$\overline{B}X := \{x : [x]_B \cap X \neq \emptyset\} \tag{2}$$

These rough set approximations form a pair of tight bounds over X based on B in the inequalities $\underline{B}X \subseteq X \subseteq \overline{B}X$. The boundary set $\tilde{B}X = \overline{B}X \setminus \underline{B}X$ [1] contains objects that we cannot say for certain to be inside or outside of X given their B attributes.

In a decision system, the family of equivalence classes generated by the decision attribute d is given by $D = U/I_{\{d\}}$. As for any set $X \subseteq U$, each equivalence class in D can be approximated by using a given attribute set B. The corresponding quality of approximation of the decision d by a set of attributes B can be measured by: $\gamma_B(D) = \frac{\sum_{X \in D} |\underline{B}X|}{|U|}$. This measure constitutes the proportion of U which can be unambiguously classified using their attribute values in B. A reduct R of A with respect to D is a minimal subset of A that preserves this approximation quality. Thus R is a reduct if $\gamma_R(D) = \gamma_A(D)$ and $B \subset R \Rightarrow \gamma_B(D) < \gamma_A(D)$. The set of reducts with respect to d in A is denoted by $Red_d(A)$.

3 Ordinal Attributes and Ordinal Decision Systems

It is quite common for attributes and decisions to assume ordinal labels like alphabetic grading systems for student's performance or Likert scales used in opinion surveys. An ordinal decision system is one in which the decision attribute is ordinal. An ordinal attribute a has an attribute value set V_a which is a linearly ordered sets of labels. We represent the label set in the form of $V_a = \{l_1^a > l_2^a > ... > l_{\perp a}^a\}$. An ordinal attribute induces a corresponding weak order $\geq_a$ (a complete and transitive relation) in U thus $\geq_a = \{(x, y) : a(x) \geq a(y)\}$. In [5] we showed that any weak order $\geq_a$ corresponds uniquely to the nested sets in the family $U_a^{\uparrow} = \{[x]_a^{\uparrow} : x \in U\}$ where $[x]_a^{\uparrow} = \{y : y \geq_a x)\}$. These nested sets correspond to the equivalence classes defined by nominal attribute values in the classic rough set theory. Thus in the case of an ordinal decision system, the induced decision order is $\geq_d$ and the corresponding nested sets is given by $U_d^{\uparrow} = \{[x]_d^{\uparrow} : x \in U\}$. We further define the following notations used in this paper.

[1] We use $X \setminus Y$ to represent difference of set X and Y.

$$[x]_a^{\downarrow} := \{y : x \geq_a y)\} \tag{3}$$

$$[x]_B^{\uparrow} = \{y : y \geq_a x, \forall a \in B\} \tag{4}$$

$$[x]_B^{\downarrow} = \{y : x \geq_a y, \forall a \in B\} \tag{5}$$

$$a^{\uparrow i} := \{x : a(x) \geq l_i^a\} \tag{6}$$

$$a^{\downarrow i} := \{x : l_i^a \geq a(x)\} \tag{7}$$

In [2], the authors consider an ordinal decision system in which all attributes, both condition and decision attributes, are ordinal. They proposed to approximate an ordinal decision system in a similarly way to approximating sets. The sets in this context are now the nested sets $d^{\uparrow i}$ and $d^{\downarrow i}$ induced by the ordinal decision. Thus for any subset of attributes $B \subseteq A$, the lower and upper approximation of these sets are given by:

$$\underline{B}d^{\uparrow i} = \{x : [x]_B^{\uparrow} \subseteq d^{\uparrow i}\} \tag{8}$$

$$\overline{B}\,d^{\uparrow i} = \{x : [x]_B^{\uparrow} \cap d^{\uparrow i} \neq \emptyset\} = \bigcup\nolimits_{x \in d^{\uparrow i}} [x]_B^{\uparrow} \tag{9}$$

$$\underline{B}d^{\downarrow i} = \{x : [x]_B^{\downarrow} \subseteq d^{\downarrow i}\} \tag{10}$$

$$\overline{B}\,d^{\downarrow i} = \{x : [x]_B^{\downarrow} \cap d^{\downarrow i} \neq \emptyset\} = \bigcup\nolimits_{x \in d^{\downarrow i}} [x]_B^{\downarrow} \tag{11}$$

The corresponding boundary sets are similarly defined:

$$\widetilde{B}d^{\uparrow i} = \overline{B}\,d^{\uparrow i} \setminus \underline{B}d^{\uparrow i} \tag{12}$$

$$\widetilde{B}d^{\downarrow i} = \overline{B}\,d^{\downarrow i} \setminus \underline{B}d^{\downarrow i} \tag{13}$$

The quality of approximation of d using the set of attributes B is then defined as:

$$\gamma_B(d) = \frac{|U \setminus \left(\left(\bigcup_i \widetilde{B}d^{\uparrow i}\right) \cup \left(\bigcup_i \widetilde{B}d^{\downarrow i}\right)\right)|}{|U|} \tag{14}$$

Reduct is then defined as any minimal subset that preserves this quality of approximation. Thus a subset $R \subseteq A$ is a reduct if and only if $\gamma_R(d) = \gamma_A(d)$ and $B \subset R \Rightarrow \gamma_B(d) < \gamma_A(d)$. In [4], we showed some problems associated with this definition of ordinal reduct and provide an alternative model to resolve these problems.

In this paper we focus on the case where in the decision system $\Delta = (U, A \cup \{d\})$, the decision attribute d is ordinal while the other attributes in A are nominal. Furthermore, similar to the approach in [4], our goal is the approximation, not of the ordinal classes, but of the underlying decision ordering $\geq_d$ generated by the ordinal decision labels in V_d. This is based on the assumption that in making an ordinal decision, the assignment of ordinal labels to objects is solely for the purpose of creating the underlying ordering $\geq_d$.

4 Approximating the Decision Ordering

Thus we regard the decision ordering as the meaning of the label assignment process and hence is more important than the actual labels assigned to the objects. To give a simple illustrate of this perspective: if we have a universe of two objects x and y, and ordinal labels 1>2>3, then the label assignments $\{x\to1,\ y\to2\}$, $\{x\to1,\ y\to3\}$, and $\{x\to2,\ y\to3\}$, will be equivalent from this perspective since they represent the same decision ordering of $x>y$. Our goal is therefore to provide a rough set approximation, in the sense of a tight upper and lower bound, for the decision ordering $>_d$ [2] base on information granules generated by the nominal attributes A. First of all, we observe that $U_d^{\uparrow} = \{[x]_d^{\uparrow} : x \in U\} = \{d^{\uparrow i}\}_i$ is the family of nested sets corresponding to the weak order $>_d$ and they are related by:

$$(>_d) = \bigcup_{X \in U_d^{\uparrow}} (X \times U \setminus X) = \bigcup_i \left(d^{\uparrow i} \times U \setminus d^{\uparrow i}\right) = \bigcup_i \left(d^{\uparrow i} \times d^{\downarrow (i+1)}\right) \tag{15}$$

This formulation provides us the means to create the upper and lower approximations for $>_d$. Since for all i and $B \subseteq A$, we have $\underline{B}\,d^{\uparrow i} \subseteq d^{\uparrow i} \subseteq \overline{B}\,d^{\uparrow i}$, thus

$$\left(\underline{B}\,d^{\uparrow i} \times \underline{B}\,d^{\downarrow (i+1)}\right) \subseteq \left(d^{\uparrow i} \times d^{\downarrow (i+1)}\right) \subseteq \left(\overline{B}\,d^{\uparrow i} \times \overline{B}\,d^{\downarrow (i+1)}\right) \tag{16}$$

provides an upper and a lower bound for the ordering generated by each level of the ordinal decision label l_i^d. Aggregating over all i's,

$$\bigcup_i \left(\underline{B}\,d^{\uparrow i} \times \underline{B}\,d^{\downarrow (i+1)}\right) \subseteq (>_d) = \bigcup_i \left(d^{\uparrow i} \times d^{\downarrow (i+1)}\right) \subseteq \bigcup_i \left(\overline{B}\,d^{\uparrow i} \times \overline{B}\,d^{\downarrow (i+1)}\right) \tag{17}$$

So the sets $\left(>_d^{\underline{B}}\right) = \bigcup_i \left(\underline{B}\,d^{\uparrow i} \times \underline{B}\,d^{\downarrow (i+1)}\right)$ and $\left(>_d^{\overline{B}}\right) = \bigcup_i \left(\overline{B}\,d^{\uparrow i} \times \overline{B}\,d^{\downarrow (i+1)}\right)$ provide respectively the lower and upper approximation to the decision ordering $>_d$. (Note: We use the same symbols for the upper and lower approximations as in [4], with the understanding that the attribute types are different and hence the approximations are also different. In the following, we will also continue to use the same symbols as in the paper for the corresponding concepts in our current formulation bearing in mind the definitions will depend on the attribute type concerned.) These are also tight bounds in as far as $\left(\underline{B}\,d^{\uparrow i}, \overline{B}\,d^{\uparrow i}\right)$ and $\left(\underline{B}\,d^{\downarrow i+1}, \overline{B}\,d^{\downarrow i+1}\right)$ are tight bounds for $d^{\uparrow i}$ and $d^{\downarrow i+1}$ respectively. The upper and lower bounds can also be written in the more familiar forms:

$$\left(x >_d^{\underline{B}} y\right) \Rightarrow (x >_d y) \tag{18}$$

$$(x >_d y) \Rightarrow \left(x >_d^{\overline{B}} y\right) \tag{19}$$

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[2] We consider approximation of $>_d$, the asymmetric part of $\geq_d$, instead of $\geq_d$. The two order relations are related by a simple formula. $>_d$ is chosen because it offers more elegant conceptual formulations.

When all d^i's are definable in $B \subseteq A$, i.e. $\underline{B}\,d^{\uparrow i} = d^{\uparrow i} = \overline{B}\,d^{\uparrow i}$, the upper and lower boundaries for the order approximation also merge and we have $\left(>_{\frac{B}{d}}\right) = \left(>_d\right) = \left(>_{\frac{\overline{B}}{d}}\right)$. The quality of ordinal approximation given $B \subseteq A$ can be measured in a similar manner as for set approximation as:

$$\gamma_B^o(d) = \frac{|>_{\frac{B}{d}}|}{|>_d|} = \frac{\left|\bigcup_i \left(\underline{B}^{\uparrow} d^{\uparrow i} \times \underline{B}^{\downarrow} d^{\downarrow i+1}\right)\right|}{|>_d|} \tag{20}$$

This measure varies between 0 and 1. It is obvious that $\gamma_B^o(d)$ will become 1 if the d^i's (the decision classes when the decision labels are treated as nominal instead of ordinal) are definable in terms of *B*. However it can be shown this is not a necessary condition for $\gamma_B^o(d)$ be equal to 1. As in the classic rough set theory, this approximation measure helps to define the concept of ordinal *d*-reduct. Thus an ordinal *d*-reduct is defined as a minimal subset of *A* which preserves this approximating quality. Formally $R \subseteq A$ is an ordinal *d*-reduct of $\Delta = (U, A \cup \{d\})$ if and only if $\gamma_R^o(d) = \gamma_A^o(d)$ and $\gamma_R^o(d) > \gamma_B^o(d),\ \forall B \subset R$. We represent the set of ordinal *d*-reducts in the context of *A* as $Red_d^o(A)$.

An alternative formulation of the ordinal *d*-reduct concept is through the concept of dispensability and independence. Under such consideration, the relation $\left(>_{\frac{B}{d}}\right)$ corresponds to the positive region in classic rough set theory. An attribute $a \in B$ is said to be ordinally *d*-dispensable if $\left(>_{\frac{B}{d}}\right) = \left(>_{\frac{B\setminus\{a\}}{d}}\right)$, otherwise it is ordinally *d*-indispensable. A set of attributes $B \subseteq A$ is said to be ordinally *d*-independent if no attribute in *B* is ordinally *d*-dispenable. Then *B* is an ordinal *d*-reduct if it is ordinally *d*-independent and $\left(>_{\frac{B}{d}}\right) = \left(>_{\frac{A}{d}}\right)$.

5 Ordinal Separability and Ordinal Discernibility Matrix

In traditionally rough set theory, a discernibility matrix can be used to compute all the reducts within a decision system. In our ordinal decision context, we can develop a similar approach to compute ordinal *d*-reducts. To do so, we need to introduce the concept of ordinal separability. Any $x, y \in U$ are ordinally separable in *B* if $x >_{\frac{B}{d}} y$ or $y >_{\frac{B}{d}} x$. Alternatively, *x* and *y* separability can be expressed in terms of their equivalence classes as $[x]_B \subseteq d^{\uparrow i}$ and $[y]_B \subseteq d^{\downarrow (i+1)}$ for some *i*, or vice versa. So *x*, *y* separability in *B* means *B* is a set of attributes which is sufficient to discriminate *x* and *y* in the decision ordering. Furthermore, if *x*, *y* are separable in $B \setminus \{a\}$ for some $a \in B$, we will say *a* is ordinally dispensable for *x*, *y* in *B*. Otherwise *a* is ordinally indispensable. A subset $R \subseteq B$ is called an ordinal reduct for *x*, *y* in *B* if they are separable in *R* and every *a* in *R* is ordinally indispensable. Thus an ordinal reduct for *x* and y is a minimal set of attributes that can discriminate them in the decision ordering. The set of ordinal d-reduct for x and y in B is denoted by $Red_d^{x,y}(B)$.

To compute the ordinal d-reduct for an ordinal decision system, we next define an ordinal discernibility matrix. For $x, y \in U$, the element c_{xy} in the ordinal discernibility matrix is defined by: $c_{xy} = Red_d^{x,y}(A)$ if x, y are ordinally separable in A, else = $\varnothing$. Thus each element of the matrix contains the minimal sets of attributes that are required to ordinally discriminate between the two object elements concerned.

From this we define an ordinal discernibility (Boolean) function:

$$f_d^o(A) = \prod_{(x,y)\in U^2} \left\{ \sum_{R\in c_{xy}} \pi(R) \right\} \text{ where } \pi(R) = \prod_{a\in R} a \tag{21}$$

Proposition

All constituents in the minimal disjunctive normal form of the function $f_d^o(A)$ are ordinal d-reducts of A.

While we will not provide our proof here, it can be observed that an ordinal d-reduct must be able to ordinally separate any pair of objects, and hence must contain at least one of the ordinal d-reduct in $Red_d^{x,y}(A)$ for any separable x, y. Therefore an ordinal d-reduct is a minimal set of attributes form from the union of selecting one member from each non-empty $Red_d^{x,y}(A)$.

We provide an example with the ordinal decision system showed as Table 1 to illustrate ordinal reducts and the use of ordinal discernibility matrix to compute them. In this example U consists of seven objects $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$. There are 4 condition attributes in $A = \{a_1, a_2, a_3, a_4\}$ which we assumed to be nominal, and an ordinal decision attribute d with ordinal labels $V_d = \{0 > 1 > 2\}$.

Table 1. An ordinal decision system

Attribute / Objects	a_1	a_2	a_3	a_4	d
x_1	1	0	2	1	1
x_2	1	0	2	0	1
x_3	1	2	0	0	2
x_4	1	2	2	1	0
x_5	2	1	0	0	2
x_6	2	1	1	0	2
x_7	2	1	2	1	1

To facilitate the computation of ordinal d-reducts for pairs of objects, we first compute the decision intervals, d-intervals, for equivalence classes of different attributes subsets. The d-interval of a set $X \subseteq U$ is the range of d values covered by the set, which will be represented by $\text{int}^d(X) = d^{\max} : d^{\min}$, where

$d^{\max} = Max_{x\in X}\{d(x)\}$ and $d^{\min} = Min_{x\in X}\{d(x)\}$. It can easily be shown that x, y are ordinally separable in B if and only if $\text{int}^d([x]_B)$ and $\text{int}^d([y]_B)$ do not intersect. Tables corresponding to each combination of attributes are created. Separability of any x, y can then be readily observed from these tables. In Table 2 we showed an example of d-interval table for $B = \{a_1, a_4\}$. It can be seen from this table that objects x_1 and x_4 are in the same equivalence class and the decision interval is 0:1. Objects x_1, x_2 are not separable in B since their corresponding decision intervals 0:1 and 1:2 overlap, and x_1, x_5 are separable as their intervals 0:1 and 2 do not intersect.

Table 2. *d*-interval table example

Attribute / Equivalence Class	a_1	a_4	*d-interval*
x_1, x_4	1	1	0:1
x_2, x_3	1	0	1:2
x_5, x_6	2	0	2
x_7	2	1	1

Table 3. The ordinal discernibility matrix

	x_1	x_2	x_3	x_4	x_5	x_6
x_2	$\varnothing$					
x_3	$\{a_3\}$ $\{a_2, a_4\}$	$\{a_3\}$ $\{a_2, a_4\}$				
x_4	$\{a_2, a_3\}$ $\{a_2, a_4\}$	$\{a_2, a_3\}$ $\{a_2, a_4\}$	$\{a_3\}$ $\{a_2, a_4\}$			
x_5	$\{a_3\}$ $\{a_1, a_4\}$ $\{a_2, a_4\}$	$\{a_3\}$ $\{a_1, a_4\}$ $\{a_2, a_4\}$	$\varnothing$	$\{a_3\}$ $\{a_1, a_4\}$ $\{a_2, a_4\}$		
x_6	$\{a_3\}$ $\{a_1, a_4\}$ $\{a_2, a_4\}$	$\{a_3\}$ $\{a_1, a_4\}$ $\{a_2, a_4\}$	$\varnothing$	$\{a_3\}$ $\{a_1, a_4\}$ $\{a_2, a_4\}$	$\varnothing$	
x_7	$\varnothing$	$\varnothing$	$\{a_3\}$ $\{a_2, a_4\}$	$\{a_2, a_3\}$ $\{a_2, a_4\}$	$\{a_3\}$ $\{a_1, a_4\}$ $\{a_2, a_4\}$	$\{a_3\}$ $\{a_1, a_4\}$ $\{a_2, a_4\}$

Using this method of judging separability, we can obtain the ordinal discernibility matrix shown as Table 3. The ordinal discernibility function for Table 3 is as follow:

$$\begin{aligned} f_d^o(B) &= (a_3 + a_2 a_4)(a_3 + a_1 a_4 + a_2 a_4)(a_2 a_3 + a_2 a_4) \\ &= (a_3 + a_2 a_4)(a_2 a_3 + a_2 a_4) \\ &= a_3 a_2 a_3 + a_2 a_4 \\ &= a_2 a_3 + a_2 a_4 \end{aligned} \tag{22}$$

where $a+b$ and ab denotes the Boolean sum and product of a and b respectively. It is obvious that after simplification we can obtain two ordinal d-reducts $\{a_2, a_3\}$ and $\{a_2, a_4\}$.

As in the classic rough set theory, we also have the concept of core attributes with respect to the ordinal decision d. The core attributes of A with respect to ordinal decision d, $CORE_d^o(A)$, are attributes which are ordinally d-indispensable in A and it can be shown that:

$$CORE_d^o(A) = \bigcap Red_d^o(A) \tag{23}$$

In the example therefore, $CORE_d^o(A) = \{a_2\}$.

6 Conclusion and Future Works

In this paper we have examined the application of rough set approach in ordinal decision making in a context where the decision attribute consists of ordinal classes while the conditional attributes are nominal as in the classic rough set theory. Taking the perspective that the ordinal label assignments in the decision is to generate an ordering of the objectives concerned, we focus our goal on approximation of this underlying ordering instead of the classes generation by label equalities. Based on this consideration, we defined the quality of approximation to be the ability of the granules of information from a set of attributes to approximate this ordering. We defined a new concept of ordinal separability and ordinal reduct for a pair of elements in the universe of discourse. From this we introduced the ordinal discernibility matrix which we used to compute ordinal reducts for an ordinal decision system.

At the moment our computation of the discernibility matrix still requires exhaustive search for all ordinal reducts for pairs of elements which is quite computationally intensive. We are working on more efficient algorithm to achieve this process. On the other hand we are testing this concept of ordinal reducts in making ordinally classifications more effective with data sets from machine learning databases.

Acknowledgement

This project is supported by the Hong Kong Polytechnic University research grant G-T667.

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