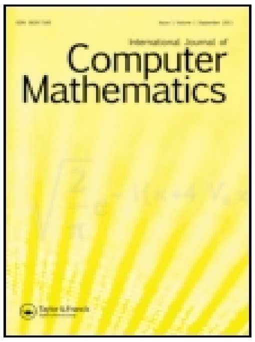

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A model of finite-step random walk with absorbent boundaries

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This paper proposes a model of finite-step lattice random walk with absorbent boundaries. We address a problem of optimal stop for this model, which is defined as the absorbent boundary value with maximum profit. Compared with many existing optimal stop investigations in the random process, our study only considers the small-sample behaviour (i.e., small number of steps behaviour) and does not consider the limit behaviour of the walk. The optimal stop time is given based on classical probability computation. Since the small-sample is more practical and common than the large-sample in many real world problems, the result obtained in this paper may provide some useful guidelines for real applications associated with the finite-step random walk such as the stock market and gambling games.

Keywords: classical probability model; small sample; lattice random walk; random walk with finite-step restriction; optimal stop of random walk

2000 AMS Subject Classification: 82B41

1. Introduction

Since Karl Pearson first presented the random walk problem in 1905, many kinds of random walk models such as simple random walk [15,10,9], symmetric random walk [15,16] and lattice random walk [9,13] have been proposed and investigated because of their wide applications to many different areas such as physics, chemistry, economics, etc. [12,1,3,7]. The simple random walk is the simplest one in which a point walks randomly along a line. Symmetric random walk means that a point walks to every direction with equal probability. The lattice random walk is such a model in which a point walks randomly on lattice in plane or space. Due to the need of real applications, Optimal Stopping Problem (OSP) for a random process, as an important issue in random walk, has attracted more and more researchers.

OSP was initiated by classical works of Wald and Wolfowitz [14] in the context of optimal stopping time based on Bayes decision procedures, and was followed by numerous publications of many authors. For example, Wolfgang Panny and Walter Katzenbeisser in paper [9] presented an approach to computing lattice paths and established a model of lattice random walk. Based on this model, some results related to distributions of random walk were derived.

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The general theory of OSPs of random processes for the case of discrete time was formulated by Snell in 1953, where one of the most interesting chapters of this theory is the so called ‘Secretary Problem’ [4,11,6,5].

OSPs of random walks were found to have a considerable number of important applications in real fields such as networks [1], stock market [3], stochastic control [7], especially in the field of gambling games [8,2].

This paper considers an old problem of a gambling game in which a player and a banker compete against each other in a series of coin tosses in which the probability that the player wins a single coin from the banker is equal to p, the probability of a loss being equal to $q = 1 - p$. Suppose that the player’s initial supply of coins is equal to N and the banker’ initial supply of coins is infinite. It is clear that the probability that the banker eventually wins all of the player’s coins is equal to 1 if the game goes on endlessly (for $p = q = 0.5$). Suppose that the player decides to play the game at most M times, that is, the player will stop playing after at most M times game or the player will stop if he/she has lost completely. What is the optimal value of M? This value is usually referred to as the optimal stop time of the game.

The process described above is usually referred to in the literature as an OSP about lattice random walk with finite steps. It seems to be a primary and simple problem of classical probability. However, due to the restriction of finite steps, no existing model is found from references to solve this problem. For solving this problem, it is difficult and inappropriate to use the characteristic functions of random variable and to use the limit theorem of random variable sequences because of the constraint of finite-step. This paper makes an attempt to initially solve this problem based on a model of lattice random walk with finite steps. The main difference between the proposed model and existing ones is that (1) the former has the restriction of finite-step whereas the later do not, and (2) for fixed M and N the limit behaviour of random walk is not considered. The key technique used in our proposed model to find the optimal stopping time is the recurrence relations. Based on the recurrence relations, the main results of the OSP for the proposed model are derived.

This paper has the following simple organization. Section 2 proposes the model of OSP mentioned above. Section 3 derives the main results of the OSP for our proposed model. Section 4 provides some simulations with respect to the results derived in Section 3, and Section 5 offers our conclusions and some issues to be studied further.

2. Problem statement

We model the above gambling game problem as a finite-step lattice random walk. As Figure 1 shows, a point in two-dimentional plane starts walking randomly from $(0, N)$ where N is a

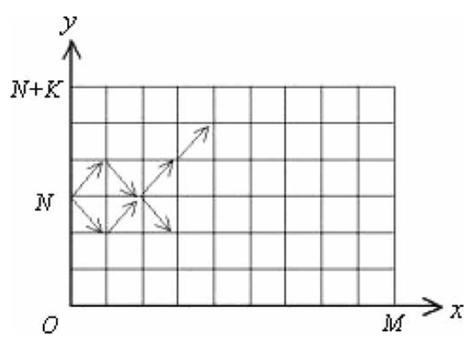

Figure 1. Random walk with finite-step restriction.

positive integer. Suppose that the current position of the point is (x, y) where x and y are two integers. Then, next time, the point will walk to $(x+1, y+1)$ or $(x+1, y-1)$ with probability 0.5 respectively. In traditional textbooks, this kind of random walk is called the lattice walk. Three straight lines, i.e., $y = 0$, $y = N + K$, and $x = M$, can be regarded as three absorbing bounds, where K and M are two positive integers. When the point walks to any one of the three absorbing bounds, the random walk stops. The process of random walking from $(0, N)$ to one of the three absorbing bounds is viewed as an experiment in the simulation.

Our main task here is to

(1) Derive the formula of the probability that the point starts at the origin $(0, N)$ and reaches the line $y = N + K$, denoted by

$$P(M, N+K, K) = \text{Probability(the point is absorbed in} Y = N+K)$$

It is easy to prove that, for fixed N and M, the function $P(M, N+K, K)$ will be decreasing with increasing of K (where the value of K can be regarded as the number of coins won by the player and $P(M, N+K, K)$ denotes the probability that the player wins K coins). We define the quantity $K \cdot P(M, N+K, K)$as the expected profit of the random walk. The optimal stop of the random walk is defined as the value of K which makes the profit function $K \cdot P(M, N+K, K)$ attain maximum.

(2) Obtain the formula of the probability that the point starts at the origin $(0, N)$ and reaches the line $y = 0$, denoted by

$$P(M, 0, K) = \text{Probability(the point is absorbed in } Y = 0)$$

$P(M, 0, K)$ can be regarded as the probability that the player lost completely.

(3) Compute the expected profit $K \cdot P(M, N+K, K)$ for $K = 1, 2, \ldots, M$ and find the optimal K such that $K \cdot P(M, N+K, K)$ achieves maximum.

(4) Derive the distribution results that the point starts at the origin $(0, N)$ and arrives at the line $x = M$.

Throughout this paper we make the following assumptions:

i) $p = q$ =0.5;

ii) if $m > n$, then$C_n^m = 0$;

iii) $P(M, N+K-a, K)$ denotes probability starting at the origin $(0, N)$ and arriving at the point $(M, N+K-a)$, for any integer a less than $N+K$.

3. Problem solving

3.1. *Main results*

We now derive the mathematical expression for the three probabilities $P(M, N+K, K)$, $P(M, 0, K)$ and $P(M, N+K-a, K)$. It is worth noting that all results obtained in this section are mainly based on recurrence relations between the numbers of lattice paths of reaching the line $x = K + 2L$ and $x = K + 2L + 2$, where $L = 0, 1, 2, \ldots, (M-K)/2 - 1$. Noting that

$$P(M, N+K, K) = \sum_{L=0}^{R} \sum_{i \geq 0} \left[\left(C_{K+2L-2}^{L-i(N+K)} - C_{K+2L-2}^{L-2-i(N+K)} \right) - \left(C_{K+2L-2}^{L-N-i(N+K)} - C_{K+2L-2}^{L-2-N-i(N+K)} \right) \right] \left(\frac{1}{2} \right)^{K+2L}$$

where $R = (M - K)/2$, we have for the special case $N \geq (M - K)/2 + 1$

$$P(M, N + K, K) = \sum_{L=0}^{R} (C_{K+2L-2}^{L} - C_{K+2L-2}^{L-2}) \left(\frac{1}{2}\right)^{K+2L}.$$

and

$$P(M, 0, K) = \sum_{L=0}^{S} \sum_{i \geq 0} \big[\big(C_{N+2L-2}^{L-i(N+K)} - C_{N+2L-2}^{L-2-i(N+K)} \big) - \big(C_{N+2L-2}^{L-K-i(N+K)} - C_{N+2L-2}^{L-2-K-i(N+K)} \big) \big] \left(\frac{1}{2}\right)^{M}$$

where $S = (M - N)/2$.

To derive the distribution expressions of reaching the line $x = M$, we need to consider the following four cases:

(1) $M - K = 2n - 1$, $M - N = 2q - 1$ where both n and q are positive integers, we have

$$P\{x = M, y = N + K - 1\} = \sum_{i \geq 0} \left[\left(C_{M-1}^{n-i(N+K)} - C_{M-1}^{n-2-i(N+K)} \right) - \left(C_{M-1}^{n-i(N+K)-N} - C_{M-1}^{n-2-i(N+K)-N} \right) \right] \left(\frac{1}{2}\right)^{M}$$

and

$$P\{x = M, y = N + K - (2a + 1)\} = \sum_{i \geq 0} \left[\left(C_{M}^{n+a-i(N+K)} - C_{M}^{n-a-1-i(N+K)} \right) - \left(C_{M}^{n+a-i(N+K)-N} - C_{M}^{n-a-1-i(N+K)-N} \right) \right] \left(\frac{1}{2}\right)^{M}$$

and

$$P\{x = M, y = 1\} = \sum_{i \geq 0} \left[\left(C_{M-1}^{q-i(N+K)} - C_{M-1}^{q-2-i(N+K)} \right) - \left(C_{M-1}^{q-i(N+K)-N} - C_{M-1}^{q-2-i(N+K)-N} \right) \right] \left(\frac{1}{2}\right)^{M}$$

$$\left(a = 1, 2, \ldots, \frac{N + K - 4}{2} \right)$$

(2) $M - K = 2n$, $M - N = 2q - 1$ where both n and q are positive integers, we have

$$P\{x = M, y = N + K\} = \sum_{i \geq o} \left[\left(C_{M-2}^{n-i(N+K)} - C_{M-2}^{n-2-i(N+K)} \right) - \left(C_{M-2}^{n-i(N+K)-N} - C_{M-2}^{n-2-i(N+K)-N} \right) \right] \left(\frac{1}{2}\right)^{M},$$

and

$$P\{x = M, y = N + K - 2a\} = \sum_{i \geq 0} \left[\left(C_{M}^{n+a-i(N+K)} - C_{M}^{n-a-i(N+K)} \right) - \left(C_{M}^{n+a-i(N+K)-N} - C_{M}^{n-a-i(N+K)-N} \right) \right] \left(\frac{1}{2}\right)^{M}$$

and

$$P\{x = M, y = 1\} = \sum_{i\geq 0}\Big[\Big(C_{M-1}^{q-i(N+K)} - C_{M-1}^{q-2-i(N+K)}\Big) - \Big(C_{M-1}^{q-i(N+K)-N} - C_{M-1}^{q-2-i(N+K)-N}\Big)\Big]\left(\frac{1}{2}\right)^{M}$$

$$\left(a = 1, 2, \ldots, \frac{M+K-3}{2}\right)$$

(3) $M - N = 2q$, $M - K = 2n - 1$ where both n and q are positive integers, we have

$$P\{x = M, y = 0\} = \sum_{i\geq 0}\Big[\Big(C_{M-2}^{q-i(N+K)} - C_{M-2}^{q-2-i(N+K)}\Big) - \Big(C_{M-2}^{q-i(N+K)-K} - C_{M-2}^{q-2-i(N+K)-K}\Big)\Big]\left(\frac{1}{2}\right)^{M}$$

and

$$P\{x = M, y = N + K - (2a+1)\} = \sum_{i\geq 0}\Big[\Big(C_{M}^{n+a-i(N+K)} - C_{M}^{n-a-1-i(N+K)}\Big) - \Big(C_{M}^{n+a-i(N+K)-N} - C_{M}^{n-a-1-i(N+K)-N}\Big)\Big]\left(\frac{1}{2}\right)^{M}$$

and

$$P\{x = M, y = N + K - 1\} = \sum_{i\geq 0}\Big[\Big(C_{M-1}^{n-i(N+K)} - C_{M-1}^{n-2-i(N+K)}\Big) - \Big(C_{M-1}^{n-i(N+K)-N} - C_{M-1}^{n-2-i(N+K)-N}\Big)\Big]\left(\frac{1}{2}\right)^{M}$$

$$\left(a = 1, 2, \ldots, \frac{N+K-3}{2}\right)$$

(4) $M - K = 2n$, $M - N = 2q$ where both n and q are positive integers, we have

$$P\{x = M, y = 0\} = \sum_{i\geq 0}\Big[\Big(C_{M-2}^{q-i(N+K)} - C_{M-2}^{q-2-i(N+K)}\Big) - \Big(C_{M-2}^{q-i(N+K)-K} - C_{M-2}^{q-2-i(N+K)-K}\Big)\Big]\left(\frac{1}{2}\right)^{M}$$

and

$$P\{x = M, y = N + K\} = \sum_{i\geq 0}\Big[\Big(C_{M-2}^{n-i(N+K)} - C_{M-2}^{L-2-i(N+K)}\Big) - \Big(C_{M-2}^{L-i(N+K)-N} - C_{M-2}^{n-2-i(N+K)-N}\Big)\Big]\left(\frac{1}{2}\right)^{M}$$

and

$$P\{x = M, y = N + K - 2a\} = \sum_{i \geq 0} \left[\left(C_M^{n+a-i(N+K)} - C_M^{n-a-i(N+K)} \right) - \left(C_M^{n+a-i(N+K)-N} - C_M^{n-a-i(N+K)-N} \right) \right] \left(\frac{1}{2} \right)^M \left(a = 1, 2, \ldots, \frac{N+K-2}{2} \right).$$

3.2. *Proofs of the main results*

Paying attention to the recurrence relations between the number of lattice paths and mathematical induction, we can prove the results listed in Section 3.1.

Without loss of generality, we assume $K \leq N$ and only give the proof for the case of $M - K = 2n, M - N = 2q$ where both n and q are positive integers. Similarly, one can prove the other cases.

Noting Figure 2, we suppose that $a_{K+2L-2,N+K-2(L-1)} (L = 0, 1, 2, \ldots, n)$ denoting the number of lattice paths from the point $(0, N)$ to the $L - th$ lattice point $(K + 2L, N + K - 2L + 2)$ on the line $x = K + 2L$ $(L = 0, 1, 2, \ldots, n)$.

Considering all those points from which the point $(K + 2L, N + K - 2L + 2)$ can be reached by the next two steps, we can obtain the following system of the recurrence relations for the number of lattice paths reaching the line $x = K + 2L$ and $x = K + 2L + 2$.

If $K + 2L - 2 < N$, then the recurrence relation is

$$\begin{pmatrix} 1 & & & & & & & & & & \\ 2 & 1 & & & & & & & & & \\ 1 & 2 & 1 & & & & & & & & \\ & 1 & 2 & 1 & & & & & & & \\ & & 1 & 2 & 1 & & & & & & \\ & & & \ddots & \ddots & \ddots & & & & & \\ & & & & 1 & 2 & 1 & & & & \\ & & & & & 1 & 2 & 1 & & & \\ & & & & & & \ddots & \ddots & \ddots & & \\ & & & & & & & 1 & 2 & 1 \\ & & & & & & & & 1 & 2 \\ & & & & & & & & & 1 \end{pmatrix} \begin{pmatrix} a_{K+2L-2,N+K-2} \\ a_{K+2L-2,N+K-4} \\ a_{K+2L-2,N+K-6} \\ a_{K+2L-2,N+K-8} \\ a_{K+2L-2,N+K-10} \\ \vdots \\ a_{K+2L-2,N+K-2m+2} \\ a_{K+2L-2,N+K-2m} \\ \vdots \\ a_{K+2L-2,N+K-2(K+L)+4} \\ a_{K+2L-2,N+K-2(K+L)+2} \\ a_{K+2L-2,N+K-2(K+L)} \end{pmatrix}$$

$$= \begin{pmatrix} a_{K+2L,N+K} \\ a_{K+2L,N+K-2} \\ a_{K+2L,N+K-4} \\ a_{K+2L,N+K-6} \\ a_{K+2L,N+K-8} \\ \vdots \\ a_{K+2L,N+K-2m} \\ a_{K+2L,N+K-2m-2} \\ \vdots \\ a_{K+2L,N+K-2(K+L)+2} \\ a_{K+2L,N+K-2(K+L)} \\ a_{K+2L,N+K-2(K+L+1)} \end{pmatrix}$$

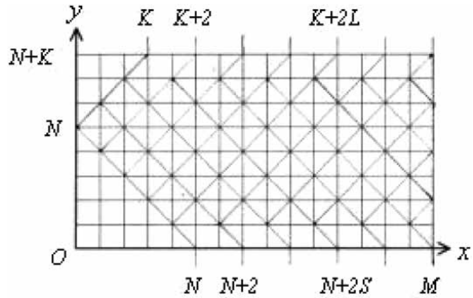

Figure 2. Recurrence relations.

where the matrix of this system has order $(K+L+2)\times(K+L)$

If $K+2L-2 \geq N$, the recurrence relation is

$$\begin{pmatrix} 1 & & & & & & & & & & \\ 2 & 1 & & & & & & & & & \\ 1 & 2 & 1 & & & & & & & & \\ & 1 & 2 & 1 & & & & & & & \\ & & 1 & 2 & 1 & & & & & & \\ & & & \ddots & \ddots & \ddots & & & & & \\ & & & & 1 & 2 & 1 & & & & \\ & & & & & 1 & 2 & 1 & & & \\ & & & & & & \ddots & \ddots & \ddots & & \\ & & & & & & & 1 & 2 & 1 & \\ & & & & & & & & 1 & 2 & \\ & & & & & & & & & 1 & 0 \end{pmatrix} \begin{pmatrix} a_{K+2L-2,N+K-2} \\ a_{K+2L-2,N+K-4} \\ a_{K+2L-2,N+K-6} \\ a_{K+2L-2,N+K-8} \\ a_{K+2L-2,N+K-10} \\ \vdots \\ a_{K+2L-2,N+K-2m+2} \\ a_{K+2L-2,N+K-2m} \\ \vdots \\ a_{K+2L-2,4} \\ a_{K+2L-2,2} \\ a_{K+2L-2,0} \end{pmatrix}$$

$$= \begin{pmatrix} a_{K+2L,N+K} \\ a_{K+2L,N+K-2} \\ a_{K+2L,N+K-4} \\ a_{K+2L,N+K-6} \\ a_{K+2L,N+K-8} \\ \vdots \\ a_{K+2L,N+K-2m} \\ a_{K+2L,N+K-2m-2} \\ \vdots \\ a_{K+2L,4} \\ a_{K+2L,2} \\ a_{K+2L,0} \end{pmatrix}$$

where the matrix of this system has order $(K+N+1)\times(K+N)$.

The numbers of lattice paths starting at the original point $(0, N)$ and leading to the points on the line $x = K$ are respectively $a_{K,N+K}, a_{K,N+K-2}, \ldots, a_{K,N+K-2r}, \ldots, a_{K,N-K}$ and $a_{K,N+K-2r} = C_K^r (r = 0, 1, 2, \ldots, K)$. Based on mathematical induction, we obtain

$$a_{K+2L-2,N+K} = \sum_{i\geq 0} \Big[\Big(C_{K+2L-2}^{L-i(N+K)} - C_{K+2L-2}^{L-2-i(N+K)} \Big) - \Big(C_{K+2L-2}^{L-i(N+K)-N} - C_{K+2L-2}^{L-2-i(N+K)-N} \Big) \Big]$$

$$
\begin{aligned}
a_{K+2L-2,N+K-2a} &= \sum_{i\geq 0}\Big[\Big(C_{K+2L}^{n+a-i(N+K)} - C_{K+2L}^{n-a-i(N+K)}\Big) \\
&\quad - \Big(C_{K+2L}^{n+a-i(N+K)-N} - C_{K+2L}^{n-a-i(N+K)-N}\Big)\Big] \\
a_{N+2L-2,0} &= \sum_{i\geq 0}\Big[\Big(C_{N+2L-2}^{L-i(N+K)} - C_{K+2L-2}^{L-2-i(N+K)}\Big) \\
&\quad - \Big(C_{N+2L-2}^{L-i(N+K)-K} - C_{N+2L-2}^{L-2-i(N+K)-K}\Big)\Big]
\end{aligned}
$$

$(a = 1, 2, \ldots, (N + K - 2)/2)$. In this way, we obtain the numbers of lattice paths from the point $(0, N)$ to the points on the line $y = N + K$. It is $a_{K-2,N+K}, a_{K,N+K}, a_{K+2,N+K}, \ldots, a_{K+n,N+K}$ which result in the following equation

$$
\begin{aligned}
P(M, N+K, K) = \sum_{L=0}^{n} a_{K+2L-2,N+K} &= \sum_{L=0}^{n}\sum_{i\geq 0}\Big[\Big(C_{K+2L-2}^{L-i(N+K)} - C_{K+2L-2}^{L-2-i(N+K)}\Big) \\
&\quad - \Big(C_{K+2L-2}^{L-N-i(N+K)} - C_{K+2L-2}^{L-2-N-i(N+K)}\Big)\Big]\left(\frac{1}{2}\right)^{K+2L}
\end{aligned}
$$

Similarly, the number of lattice paths from the point $(0, N)$ to the points on the line $y = 0$ are respectively $a_{N-2,0}, a_{N,0}, a_{N+2,0}, \ldots, a_{N+q,0}$ and therefore we have

$$
\begin{aligned}
P(M, 0, K) = \sum_{L=0}^{q} a_{N+2L-2,0} &= \sum_{L=0}^{q}\sum_{i\geq 0}\Big[\Big(C_{N+2L-2}^{L-i(N+K)} - C_{N+2L-2}^{L-2-i(N+K)}\Big) \\
&\quad - \Big(C_{N+2L-2}^{L-K-i(N+K)} - C_{N+2L-2}^{L-2-K-i(N+K)}\Big)\Big]\left(\frac{1}{2}\right)^{N+2L}
\end{aligned}
$$

The distribution law that the point starts at the origin $(0, N)$ and arrives at the line $x = M$ $(M = K + 2L)$ is

$$
\begin{aligned}
P_{M,N+K-2a} &= \sum_{i\geq 0}\Big[\Big(C_{M}^{n+a-i(N+K)} - C_{M}^{n-a-i(N+K)}\Big) \\
&\quad - \Big(C_{M}^{n+a-i(N+K)-N} - C_{M}^{n-a-i(N+K)-N}\Big)\Big]\left(\frac{1}{2}\right)^{M}
\end{aligned}
$$

$(a = 1, 2, \ldots, (N + K - 2)/2)$. It is our required equation.

3.3. *Finding optimal stopping time K*

For any fixed M and N, there will be M values of $KP(M, N + K, K)$ in all while the integer K changes from 1 to M. Hence, from M values of $KP(M, N + K, K)$ we can find the maximum and its corresponding optimal K (which makes the quantity $KP(M, N + K, K)$ attain maximum).

And $\max_K(KP(M, N + K, K))$ denotes the maximum expected profit of optimal stopping time. We can consider the profit function

$$\text{Profit}\,(K) = K \cdot P(M, N + K, K)$$

for $K \in [0, M]$ and then analyse its maximum. However, it is rather difficult to find an analytic expression for the optimal stopping time due to the complexity of the equation. Since the $P(M, N + K, K)$ has been derived explicitly, we can use the exhausted search to find the optimal K for fixed M and N. Section 4 gives some simulations for fixed M and N to explore the changing tendency of the profit function with the increase of K from 1 to M. It is interesting to view from Figures 3–6 that the profit functions for diverse M and N have a similar shape.

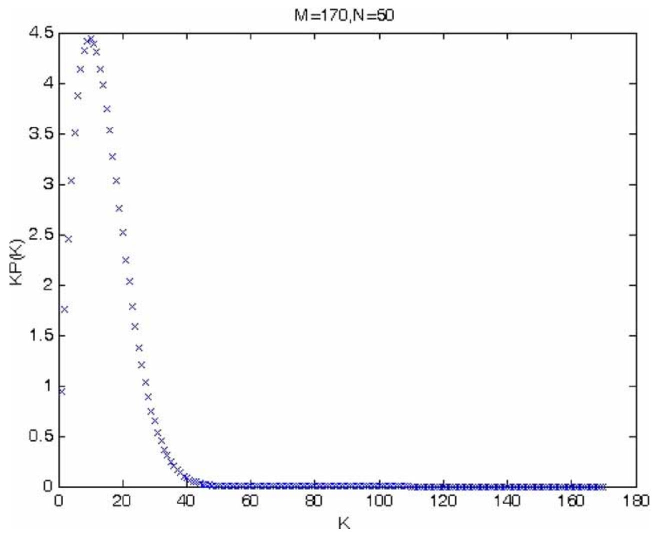

Figure 3. Expected profit with $M = 170$, $N = 50$.

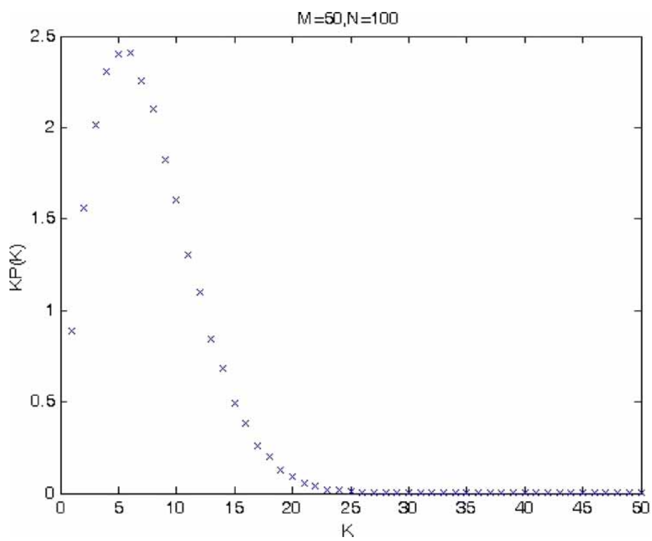

Figure 4. Expected profit with $M = 50$, $N = 100$.

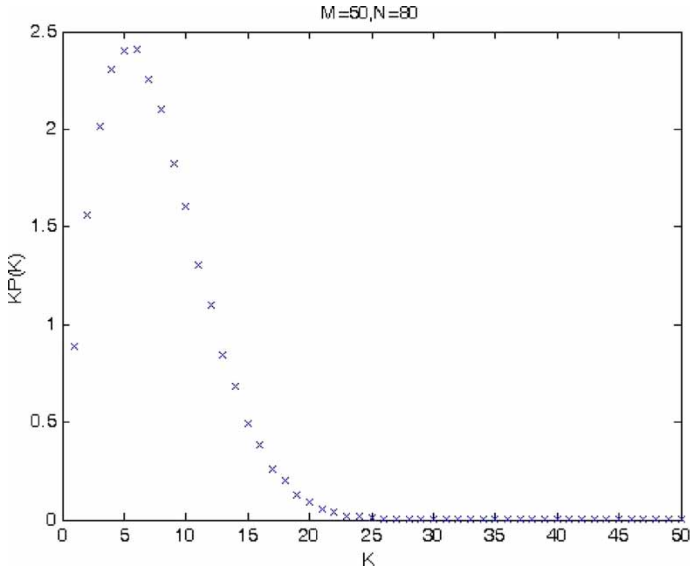

Figure 5. Expected profit with $M = 50$, $N = 80$.

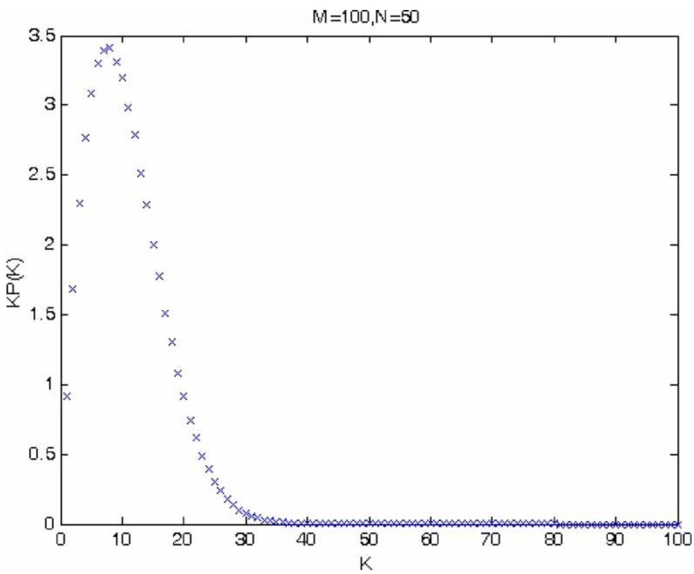

Figure 6. Expected profit with $M = 100$, $N = 50$.

4. Simulations

A lot of simulation experiments on the change of $KP(M, N + K, K)$ with the increase of K from 1 to M for diverse and fixed M and N have already been conducted. Using exhausted search, we can find the optimal K which makes the profit function achieve maximum. From Figures 3–6, one can observe changing tendency of the profit function $KP(M, N + K, K)$ with the increase of K.

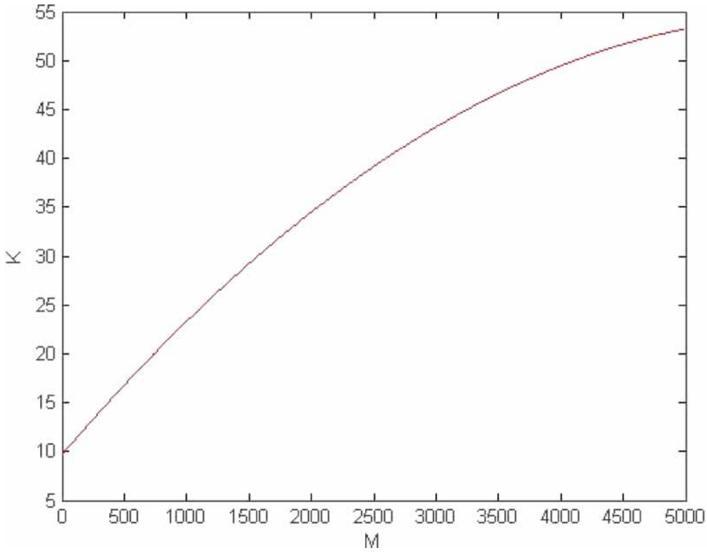

Figure 7. An approximate relation between K and M.

We briefly analyse Figures 3–6. Figure 3 is the profit function $KP(M, N+K, K)$ with $M = 170$, $N = 50$. When K is equal to 9, the expected profit achieves maximum. In Figure 4, M is 50, N is 100, and the optimal K is 5. Figure 5 is with $M = 50$, $N = 80$, and optimal K is equal to 5. Figures 4 and 5 are the same and have the identical optimal K, because both of them satisfy $N \geq (M-K)/2 + 1$ for the all $K(K = 1, 2, \ldots, M)$. They belong to the special case in Section 3.1.

Figure 6 is with $M = 100$, $N = 50$. When K is equal to 7, the maximum expected profit is achieved. It may provide such a guideline that, if the player has 50 coins and the probability of wining or losing a coin is 0.5 and he plays at most 100 times, then the optimal time is the 7th.

With the increasing of M, the optimal K becomes increasing too. But the increasing amount of the optimal K is relatively smaller when M increases larger. It is clear that the random walk model with finite step restriction will switch to the model of random model in common sense if M goes to infinity.

It is useful and interesting to further investigate the relationships among the M, N and K.

Now we initially observe the relation between $K^* = \arg\mathrm{Max}_K\ (K \cdot P(M, N+K, K))$ and M when N is considered as a given constant. Since the exact expanded expression of $(K \cdot P(M, N+K, K))$ cannot be found, it is difficult to obtain a formula to describe the dependence of K and M for given N. Hence we give an experimental demonstration regarding how K^* depending on M. See Figure 7. It shows an increasing phenomenon that K^* increases when M increases.

5. Conclusions and future works

This paper proposes a new model of finite-step lattice random walk with absorbent boundaries and presents some properties for the model. Based on this model, a type of the optimal stopping time is defined and derived. This model is expected to provide some useful guidelines on the viewpoint of maximum profit for related applications such as the stock market and gambling games. So far, the results given and the tools used in this paper are elementary. Many issues related to this model need to be studied further, for example, the following:

(1) the case that p is not equal to q;
(2) the extension from single step walk model to the model of walk distribution such as $\begin{pmatrix} 0 & N/4 & 2N/4 & 3N/4 & N \\ 0.05 & 0.15 & 0.6 & 0.15 & 0.05 \end{pmatrix}$; and
(3) the analytic expression of the optimal K.

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