EDITORIAL



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Published online: 30 March 2020 © Springer-Verlag GmbH Germany, part of Springer Nature 2020

Three-way decision (3WD) was proposed by Yiyu Yao to solve learning and reasoning problems by means of ternary classification. It trisects a universe of discourse and represents the uncertainty in the form of non-commitment. Granular computing (GrC) is an effective theory for information granulation. As far as we know, some well-known experts made substantial contributions in the early development of GrC such as Lotfi A. Zadeh, Tsau Young Lin, Yiyu Yao, Witold Pedrycz, etc. Concept lattice (CL), presented by Rudolf Wille in studying formal concept analysis, was to make data analysis based on Boolean relation. Up to now, 3WD, GrC and CL have been used to develop different types of machine learning models. For instance, 3WD-based clustering, multi-granularity structure learning and clustering, and concept-cognitive learning model for classification.

In recent years, there have been appearing a large number of new promising research directions in integrating 3WD, GrC and CL together, such as 3WD (GrC, CL) based conflict analysis and cognitive computing, multi-granularity incremental three-way concept learning, approximation updating and attribute reduction, multi-granularity sequential clustering, decision making and active learning, and so on. These new interdisciplinary researches have been promoting the development of 3WD, GrC and CL, and they are very welcome in the fields of granular computing and rough set theory.

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² Faculty of Science, Kunming University of Science and Technology, Kunming 650500, Yunnan, People's Republic of China This issue aims to provide a snapshot of novel ideas and new challenging problems in integrating 3WD, GrC and CL. It consists of 14 papers which are briefly summarized as follows.

Yiyu Yao uses tri-level thinking to represent the existing models of 3WD, and gives basic issues and components of tri-level thinking as well as a perception-cognition-action tri-level conceptual model for the purpose of explaining how to use or implement three-way decision in related intelligent systems. The tri-level thinking in his paper entitled "Trilevel thinking: models of three-way decision" can be beneficial for readers to well understand the mechanism of 3WD and is accelerating the development of 3WD.

In the second paper entitled "A comparison study of optimal scale combination selection in generalized multi-scale decision tables", Wei-Zhi Wu and Yee Leung put forward different scale combinations, clarify their relationship, and discuss the problem of selecting an optimal scale combination from a generalized multi-scale decision table.

Bo Yang and Jinhai Li construct four types of complex networks based on the collected dataset of 3WD articles to show the relationships between authors, affiliations, papers and keywords in three-way decision researches. The obtained results in their paper entitled "Complex network analysis of three-way decision researches" provide answers to the following questions: which authors and affiliations actively promote the development of 3WD, which papers are influential in the domain of 3WD, and what are the hottest research topics in 3WD.

In the fourth paper entitled "Three-way decisions: beyond rough sets and granular computing", Dun Liu, Xin Yang and Tianrui Li make a systematical review of important developmental stages of 3WD, including the explanation of 3WD with "multi-level" and "multi-view" granular computing strategies, and a comprehensive analysis of several improved versions of 3WD via multi-level and multi-view structures.

In the fifth paper entitled "An efficient three-way clustering algorithm based on gravitational search", Hong Yu,



Zhihua Chang, Guoyin Wang and Xiaofang Chen improve the existing three-way clustering algorithms by means of gravitational search to adjust thresholds dynamically for the purpose of achieving better clustering performance.

Ming-Wen Shao, Meng-Meng Lv, Ken-Wen Li and Chang-Zhong Wang investigate attribute-oriented and object-oriented multi-granularity concept lattices based on granularity trees of attributes in their paper entitled "The construction of attribute (object)-oriented multi-granularity concept lattices". More specifically, attribute-oriented and object-oriented multi-granularity concept lattices can be transformed effectively from coarse granularity to fine granularity or fine granularity to coarse granularity. It is also observed that the change of granularity is due to multiple values of attributes under a multi-level or multi-scale environment.

Fan Min, Shi-Ming Zhang, Davide Ciucci and Min Wang discuss three-way active learning algorithm through clustering selection. The main contribution of their paper entitled "Three-way active learning through clustering selection" lies on the coarse-to-fine idea coming from granular computing and instance processing by three-way thinking. The advantage of the proposed algorithm is that the appropriate clustering can be selected flexibly in the implementation of active learning.

Xiuyi Jia, Ya Rao, Lin Shang and Tongjun Li put forward a heuristic attribute reduction algorithm for decision tables by taking into account both intra-class similarity and inter-class similarity of objects in their paper entitled "Similarity-based attribute reduction in rough set theory: a clustering perspective". The basic thinking behind this strategy is that the discriminating ability of clusters will probably be increased by gradually excluding redundant attributes from a decision table in which decision classes are viewed as clusters.

In the ninth paper entitled "Using single axioms to characterize *L*-rough approximate operators with respect to various types of *L*-relations", Bin Pang and Ju-Sheng Mi present several kinds of *L*-rough approximate operators based on different *L*-relations and their compositions, and further prove that single axiom can be found to characterize a certain form of *L*-rough approximate operator axiomatically.

In the tenth paper entitled "A general conflict analysis model based on three-way decision", Guangming Lang proposes a general framework for three-way conflict analysis, and shows that the existing five conflict analysis models can be viewed as special cases of the proposed framework.

In the eleventh paper entitled "Optimal scale selection by integrating uncertainty and cost-sensitive learning in multi-scale decision tables", Xueqiu Zhang, Qinghua Zhang, Yunlong Cheng and Guoyin Wang reconsider the issue of selecting optimal scale by simultaneously minimizing the uncertainty, test cost and delay cost. It should be pointed out that setting appropriate values for parameters in the proposed algorithm is very important and the interpretability of the final results is sensitive to these parameters.

Yanyan Yang, Shiji Song, Degang Chen and Xiao Zhang discuss the issue of incremental feature selection for heterogeneous data based on discernible neighborhood counting in their paper entitled "Discernible neighborhood counting based incremental feature selection for heterogeneous data". Informative feature addition and redundant feature removal are two basic strategies for the design of incremental feature selection algorithm.

Di Dai, Huaxiong Li, Xiuyi Jia, Xianzhong Zhou, Bing Huang and Sunning Liang give a co-training approach for sequential three-way decisions. Different from the traditional co-training models, the proposed co-training approach in their paper entitled "A co-training approach for sequential three-way decisions" takes confidence into account except the probability of sample labeling used in the existing work.

The final paper entitled "Knowledge granularity based incremental attribute reduction for incomplete decision systems", written by Chucai Zhang, Jianhua Dai and Jiaolong Chen, analyzes the incremental mechanism of updating reducts when multiple objects are added into or removed from an incomplete decision system. In addition, knowledge granularity and uncertainty measure are also defined to design the required heuristic information.

In a word, this issue is providing some useful references and guidelines for those who are interested in novel ideas and new challenging problems in integrating three-way decision, granular computing and concept lattice, and particularly for those who are trying to develop data-driven machine learning approaches based on 3WD, GrC and CL.

We would also like to take this opportunity to thank both authors and reviewers for their great efforts and supports to this issue.

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